

# Channel Model for Simulating Data Transmission on Terrestrial FSO Paths

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**Abstract.** The paper deals with a model for simulating atmospheric Free-Space Optical (FSO) channel. Due to the effect of atmospheric turbulence the channel is characterized by signal fluctuations in the order of tens of milliseconds, which are several orders of magnitude longer than the duration of packets being transmitted. For moderate transmission rates the FSO channel can be simply modeled as a channel with slowly varying attenuation. It allows computing the probability of the occurrence of errors in each packet by means of analytical formulae, i.e. the simulation event rate is derived from the packet rate rather than from the bit rate.

**Keywords:** Free-space optical links, atmospheric turbulence, modeling.

## 1 Introduction

The technology of Free-Space Optical (FSO) links consists in transmitting information by means of light beams in space or in the atmosphere. At present, terrestrial systems of up to 1.25 Gbs with a range of up to 2 km are commercially available. The majority of the links are designed as simple protocol-independent repeaters on the physical layer, using on-off keying (OOK) of laser diode or LED in the transmitter. Other types (ground-space, e.g.) are in experimental stage [1].

The atmosphere causes very long outages (longer than an hour) and relatively short outages in the millisecond scale. The long outages are the main phenomena influencing short-range links. Statistical data for large territories can be obtained from long-term meteorological observations [2]. The link availability depends on the value of the fade margin. With respect to safety regulations and available components, carrier-grade availability can be obtained only for very short links ( $\approx 100$  m for the temperate climate) [3] or in combination with a microwave link [4].

Long-range terrestrial links with a tight power budget are, in addition, influenced by atmospheric turbulence. Inhomogeneity in the atmosphere cause power fluctuations at the receiver in the millisecond time scale for static terminals and in the microsecond time scale for aircraft. These short outages increase the bit-error rate and interfere with communication protocols. General theory of turbulence can be found in the classical book [5]. Theoretical information capacity of the channel for different scintillation models was analyzed in [6]. Many practical experiments for terrestrial and ground-space links have been conducted at DLR [1], [7], [9].

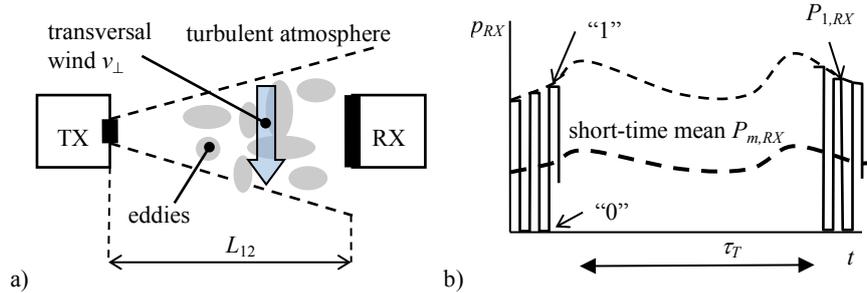
A millisecond outage results in the loss of hundreds or thousands of packets for high-speed networks. The significant difference between the packet duration and the “period” of atmospheric turbulence allows using analytic formulae for the calculation of packet error probability. The simulation event rate is then derived from the packet rate rather than from the bit rate.

The paper provides a simple framework for FSO statistical channel modeling (Sections 2) and the simulation of packet transmission (Sections 3).

## 2 FSO Channel Model

### 2.1 Statistics of Received Power

Let us consider the OOK FSO terrestrial link in Fig. 1a. There are two main mechanisms influencing the optical beam – scattering on water droplets and atmospheric turbulence. Attenuation caused by scattering increases significantly during fog, rain, and snowfall and may cause a long outage. It is a very slow process, which determines the overall availability of link [2]. In thick fog the attenuation can reach hundreds of dB/km in fog [4].



**Fig. 1.** a) FSO link configuration; b) Received optical power (ON-OFF keying used).

With respect to channel linearity the received optical power can be expressed in decibels as

$$p_{RX}(t) = p_{TX}(t) - \alpha_{SYS} - \alpha_S(t) - \alpha_T(t) \text{ [dB]}, \quad (1)$$

where  $\alpha_{SYS}$  represents all constant losses including the free-space propagation loss,  $\alpha_S$  is the attenuation caused by scattering, and  $\alpha_T$  is the attenuation caused by atmospheric turbulence. Atmospheric turbulence just redistributes energy in the beam, i.e. there is no energy loss. If the receiving aperture is smaller than the beam cross-section, the received power fluctuates, Fig. 1b. This is expressed by the time-varying apparent attenuation  $\alpha_T$ .

Equation (1) comprises three processes with very different time scales, which can be described by the relation

$$\tau_{DATA} \ll \tau_T \ll \tau_S, \quad (2)$$

where  $\tau_{DATA}$  represents the duration of transmitted symbols, and  $\tau_T$  and  $\tau_S$  are “periods” of atmospheric turbulence and scattering, respectively. Typical values are  $\tau_{DATA} < 1\mu\text{s}$ ,  $\tau_T \approx 1\text{ms} - 10\text{ms}$ ,  $\tau_S > 1\text{min}$ .

Considering the equal probability of the symbols “0” and “1” and with respect to (2) we can define the short-time mean power at the receiver as

$$P_{m,RX} = 0.5 P_{1,RX} , \quad (3)$$

which fluctuates randomly due to atmospheric turbulence.  $P_{1,RX}$  is the received power for symbol “1”, see Fig. 1b. The normalized received power is then

$$P_N = \frac{P_{m,RX}}{\langle P_{m,RX} \rangle} , \quad (4)$$

where the mean value  $\langle P_{m,RX} \rangle$  is computed over the time scale of turbulences, i.e.  $\alpha_S$  is assumed to be constant.

The *Power Scintillation Index* (PSI)

$$\sigma_p^2 = \frac{\langle P_{m,RX}^2 \rangle - \langle P_{m,RX} \rangle^2}{\langle P_{m,RX} \rangle^2} = \langle P_N^2 \rangle - 1 \quad (5)$$

provides a measure of the scintillation strength. Weak fluctuations are characterized by  $\sigma_p^2 < 1$ . The scintillation index is inversely proportional to the diameter  $D$  of receiving aperture due to the effect of averaging. Atmospheric turbulence reduces the beam spatial coherence, which is characterized by the coherence length (for weak and moderate turbulence)

$$\rho_C \approx 0.40 \sqrt{L_{12} \lambda} , \quad (6)$$

where  $\lambda$  is the wavelength, and  $L_{12}$  is the distance. The larger the receiver aperture the more uncorrelated contributions are summed at the photodetector [5]. The effect can be expressed as

$$\frac{\sigma_p^2(D_1)}{\sigma_p^2(D_2)} \approx \frac{A(D_1)}{A(D_2)} , \quad (7)$$

where  $D_1$  and  $D_2$  are two diameters of the receiving aperture and  $A(D)$  is the aperture-averaging factor

$$A(D) = \left[ 1 + 0.333 \left( \frac{\pi D^2}{2L_{12} \lambda} \right)^{5/6} \right]^{-7/5} . \quad (8)$$

On the assumption of weak fluctuations and a small receiving aperture ( $D < 5\rho_C$ ),  $P_N$  is log-normally distributed with the probability density function [5]

$$f_{P_N,w}(p) = \frac{1}{p\sigma_L\sqrt{2\pi}} \exp\left(-\frac{\left[\ln p + \frac{1}{2}\sigma_L^2\right]^2}{2\sigma_L^2}\right), \quad (9)$$

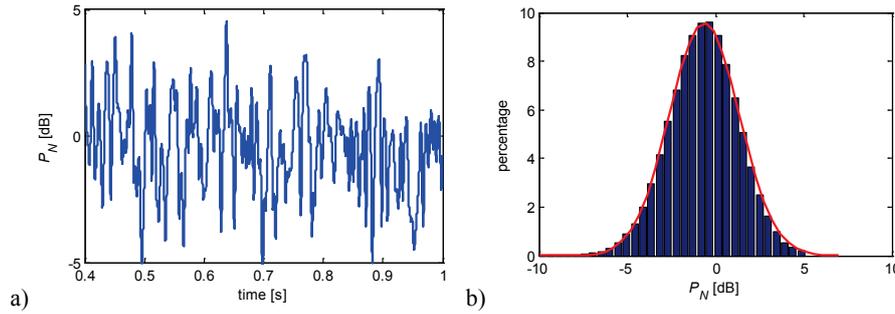
where

$$\sigma_L = \sqrt{\ln(\sigma_p + 1)}. \quad (10)$$

For moderate and strong fluctuations a number of models have been proposed. The Gamma-Gamma distribution is convenient as its parameters can be obtained both theoretically and experimentally. The PDF is given as

$$f_{P_N,s}(p) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} p^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta p}), \quad (11)$$

where  $K_a(\cdot)$  is the modified Bessel function of the second kind of order  $a = \alpha - \beta$ ,  $\Gamma$  is the Gamma function, and  $\alpha$  and  $\beta$  are the effective numbers of small-scale and large-scale eddies of the turbulent environment [6].



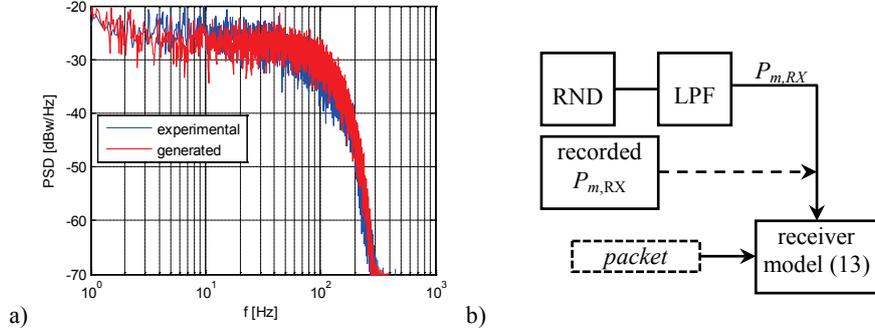
**Fig. 2.** a) Normalized optical power; b) Lognormal fit of 1 s period.

Fig. 2 shows the normalized power for  $\sigma_p^2 = 0.23$  and its lognormal fit for an experimental link with following parameters:  $\lambda = 1550\text{nm}$ ,  $L_{12} = 500\text{m}$ ,  $D = 25\text{mm}$ ,  $R = 125\text{Mbs}$  [9]. The sampling frequency was 10 kHz with a 2 kHz anti-aliasing filter.

## 2.2 Frequency-Domain Behavior

The probability density function of received power does not describe the temporal behavior of the channel, which is crucial for time-domain simulation. Using the Taylor principle of *frozen turbulence*, the “frequency” of scintillations is proportional to the transversal wind speed. The *eddies* in Fig. 1a can be thought of as wafted by the transversal component  $v_{\perp}$  of wind (or by relative speed of mobile terminal) [5]. The average fade duration is proportional to the ratio

$$\tau \approx \frac{\rho_C}{v_{\perp}} . \quad (12)$$



**Fig. 3.** a) PSD of normalized power; b) Block diagram of channel simulation (RND – lognormal random number generator, LPF – low-pass filter).

Fig. 3a shows the power spectral density of experimental link from section 2. As can be seen the main energy component is contained within a band of 100 Hz. The “bandwidth” scales with (12).

The time-domain representation of the short-time mean power  $P_{m,RX}$  can be approximately simulated connecting a random number generator with a low-pass filter. Its transfer function depends on the desired PSD. In case of the experimental link, an equiripple FIR filter with sampling frequency 10 kHz was used ( $f_p = 10\text{Hz}$ ,  $f_s = 300\text{Hz}$ ,  $A_p = 3\text{dB}$ ,  $A_s = 60\text{dB}$ ). Fig. 3a shows both experimental and generated spectra.

### 3 Simulation of Packet Transmission

The inequality (2) greatly simplifies the channel modeling. Considering transmission rates of 100Mb/s and above, an interval of 100 $\mu\text{s}$ , during which the received power is practically constant, corresponds to a block of more than 10<sup>4</sup> bits. Bit error probability during the interval depends on constant signal-to-noise ratio in the receiver [8]. Let  $p_b(P_{m,RX})$  be the probability of an independent single bit error depending on slowly varying short-time mean power (3), which characterizes the receiver. Then the probability of  $k$  errors occurring in a block of  $n$  bits is given by the Binomial distribution

$$P_n(k) = \binom{n}{k} p_b^k (1 - p_b)^{n-k} . \quad (13)$$

The instant value of short-time mean power  $P_{m,RX}$  can be obtained by sampling the recorded or generated time-domain signal, see Fig. 3b at the center of the packet interval. It allows computing the probability of the occurrence of errors in each packet by means of analytical formulae, i.e. the simulation event rate is derived from the packet rate rather than from the bit rate.

## 4 Conclusions

The paper presents a simple model for simulating the FSO channel, using samples of optical power received from a real link or generated by a turbulence model. Provided formulae (7), (8), and (12) allow a simple transformation of model parameters to a different scenario.

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### References

- [1] H. Henniger, B. Epple, D. Giggenbach, Mobile FSO Activities in Europe and Fading Mitigation Approaches. *Proc. 17th Int'l. Conf. Radioelektronika*, 2007, pp. 1-6.
- [2] Z. Kolka, O. Wilfert, O. Fiser, Achievable qualitative parameters of optical wireless links. *J. Optoelect. Adv. Mat.*, vol. 9, no. 8, August 2007, pp. 2419 – 2423.
- [3] I. I. Kim, B. McArthur and E. Korevaar, Comparison of laser beam propagation at 785 nm and 1550 nm in fog and haze for optical wireless communications. *Proc. of SPIE*, vol. 4214, 2001, pp. 26-37.
- [4] F. Nadeem, V. Kvicera, M. S. Awan, E. Leitgeb, S. S. Muhammad, G. Kandus, Weather Effects on Hybrid FSO/RF Communication Link. *IEEE J. Sel. Areas in Comm.*, vol.27, no.9, Dec. 2009, pp. 1687-1697.
- [5] L.C. Andrews, C.Y. Phillips, J. Hopen, *Laser Beam Scintillation with Applications*. SPIE Press, Bellingham (WA), USA, 2001.
- [6] M. Uysal, J. Li, M. Yu, Error Rate Performance Analysis of Coded Free-Space Optical Links over Gamma-Gamma Atmospheric Turbulence Channels. *IEEE Trans. on Wireless Comm.*, vol. 5, no. 6, 2006, pp. 1229 – 1233.
- [7] H. Henniger, A. Gonzalez, Transmission Scheme and Error Protection for Simplex Long-Distance Atmospheric FSO Systems. *Mediterranean Journal of Electronics and Communications*, vol. 2, no. 3, 2006, pp. 118-126.
- [8] N. Perlot, Evaluation of the scintillation loss for optical communication systems with direct detection. *Optical Engineering*, vol. 46, no. 2, 2007, pp. 025003.
- [9] M. Kubicek, H. Henniger, Z. Kolka, Bit error distribution measurements in the atmospheric optical fading channel. *Proc. of SPIE*, vol. 6877, 2008.